Advanced Algorithms

March 3, 2026

Logistics

- Welcome Dr. Dave!
- Exercise set 4 due
- Thank you for submitting feedback
- Exercise set 5 released tonight, due next week as usual

Where we are

1. Linear Prog. can model many interesting problems, and is in P

2. Integer Prog. can model MANY problems but is NP-hard

3. We can use Linear Prog. to tackle certain Integer Programs

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*Also, a *language* of optimization

- You are running a Hot Tub production company.
- You can produce two types of hot tubs: Aqua-Spas and Hydro-Luxes.
- They require resources (pumps, labor, and tubing), and yield a certain profit

	Aqua-Spa	Hydro-Lux
Pumps	*1*	*1*
Labor	*9 hours*	*6 hours*
Tubing	*12 feet*	*16 feet*
Price	*$350*	*$300*

- You have 200 pumps, 1566 hours of labor, and 2880 feet of tubing.
- How many of each hot tub to produce if we want to maximize sales?

Integer Programming formulation:

x = number of Aqua-Spas to produce
y = number of Hydro-Luxes to produce

Maximize: $350x + 300y$

Subject to:

$x + y \leq 200$	(pumps)
$9x + 6y \leq 1566$	(labor)
$12x + 16y \leq 2880$	(tubing)
$x, y \geq 0$	(non-negativity)
x,y integer	

Linear Programming relaxation:

x = number of Aqua-Spas to produce
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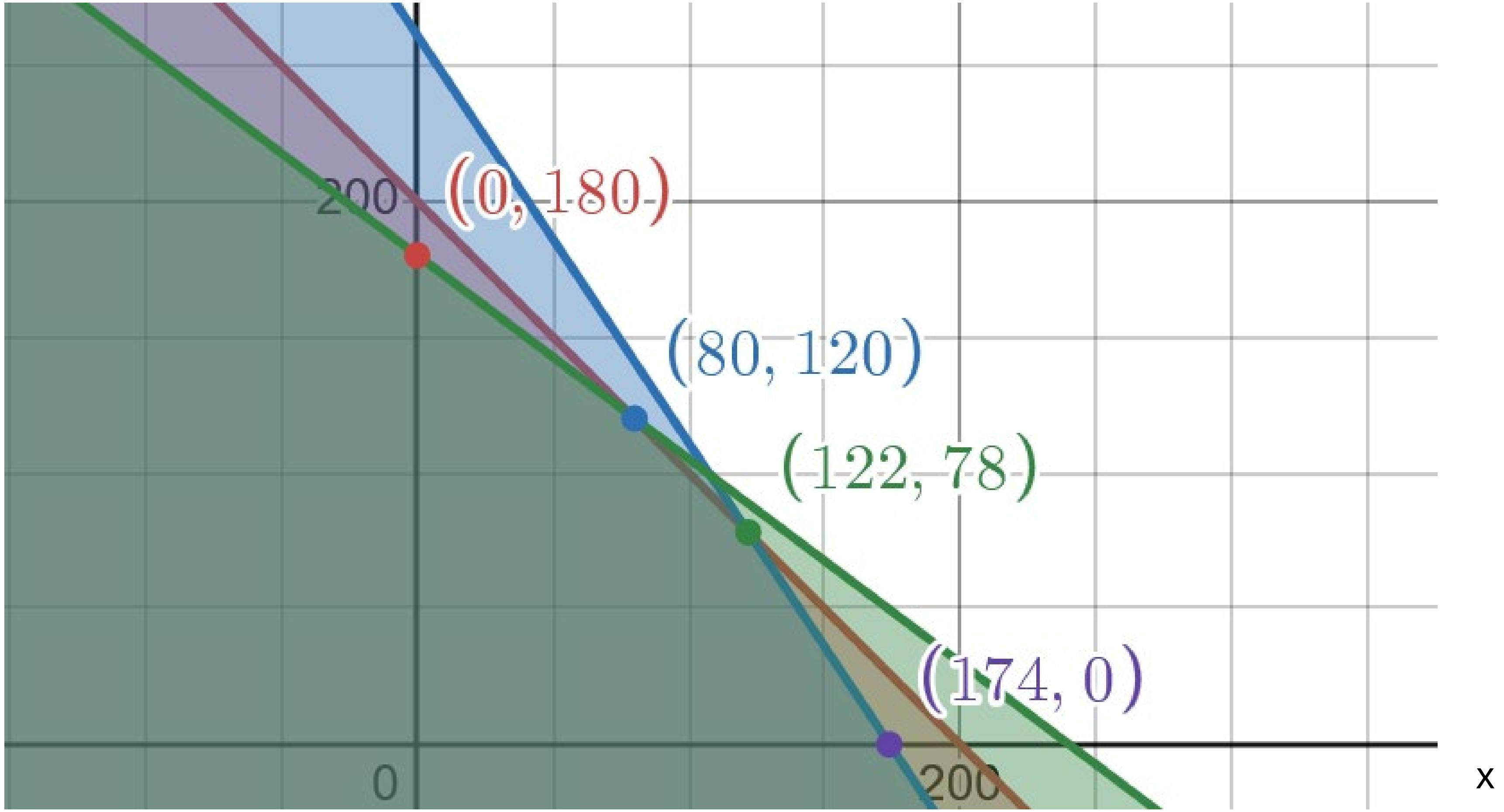
Every Integer Program has an LP relaxation

How can this be used in an algorithm?

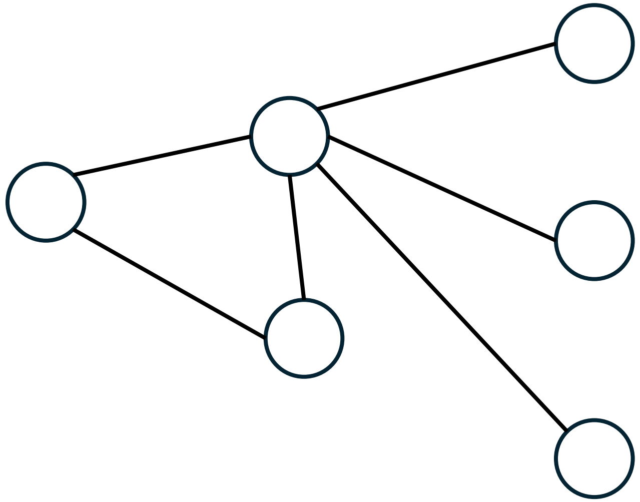
Hot Tub LP



Vertex Cover

- Given an undirected graph $G = (V, E)$, find a vertex cover $S \subseteq V$ of **minimum size**

- A **vertex cover** is a set $S \subseteq V$ such that for every edge $(u, v) \in E$, either $u \in S$ or $v \in S$.



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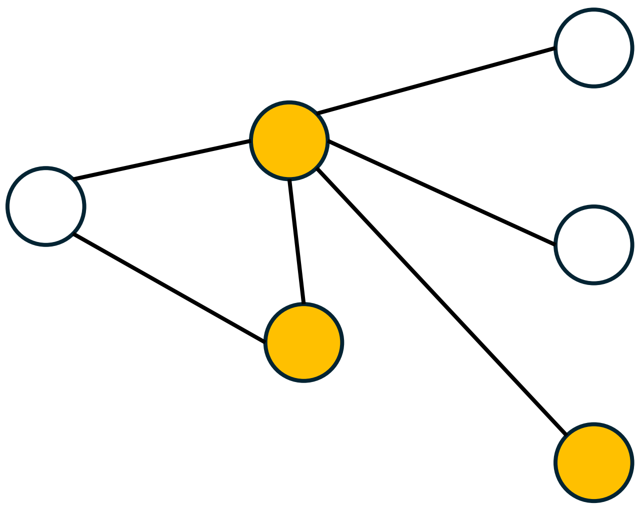
- (This is NP-hard)
- Integer Programming formulation?



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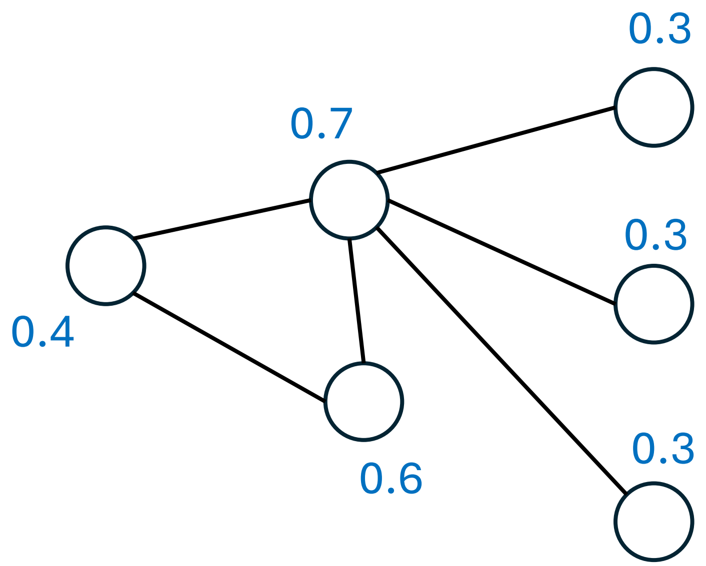
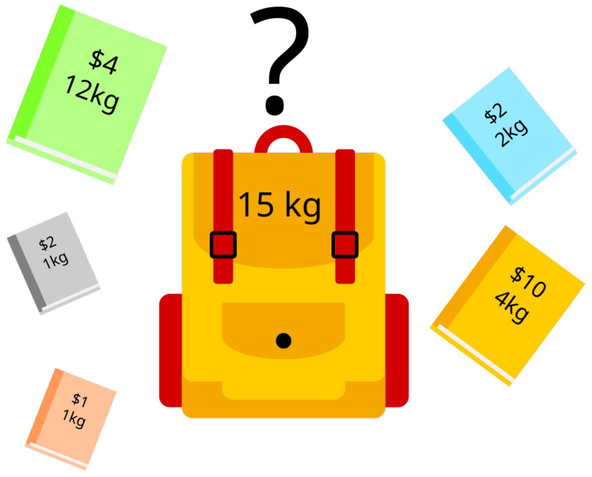


Knapsack



Given:

- a set of n items with weights w_i and values v_i
- a knapsack with total capacity W

Find a set of items S with maximum value that fits in the knapsack

(Also NP-hard)

Integer Programming formulation?

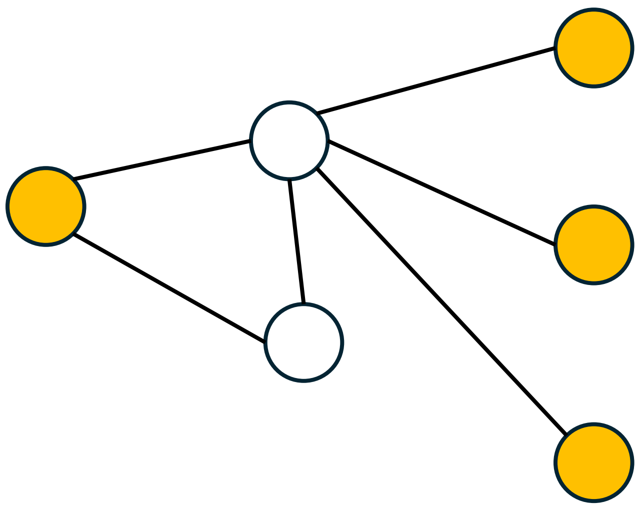
Independent Set

- Given an undirected graph $G = (V, E)$, find independent set $S \subseteq V$ of **maximum size**

- An **independent set** is $S \subseteq V$ such that no edge has both endpoints in S

- (This is NP-hard)

- Integer Programming formulation?



Consider the following

Given a bipartite graph $G = (L \cup R, E)$, define the following Linear Program:

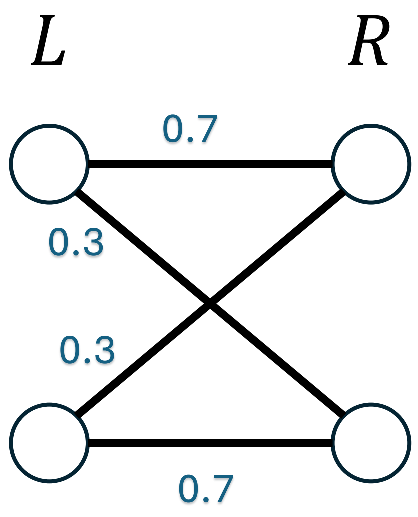
$$\min \sum_{e \in E} c_e x_e$$

such that:

$$\sum_{e \in \delta(u)} x_e = 1 \qquad \text{for all } u \in L$$

$$\sum_{e \in \delta(v)} x_e = 1 \qquad \text{for all } v \in R$$

$$x_e \geq 0 \qquad \text{for all } e \in E$$



What are integer solutions to this LP?